

## Appendix H

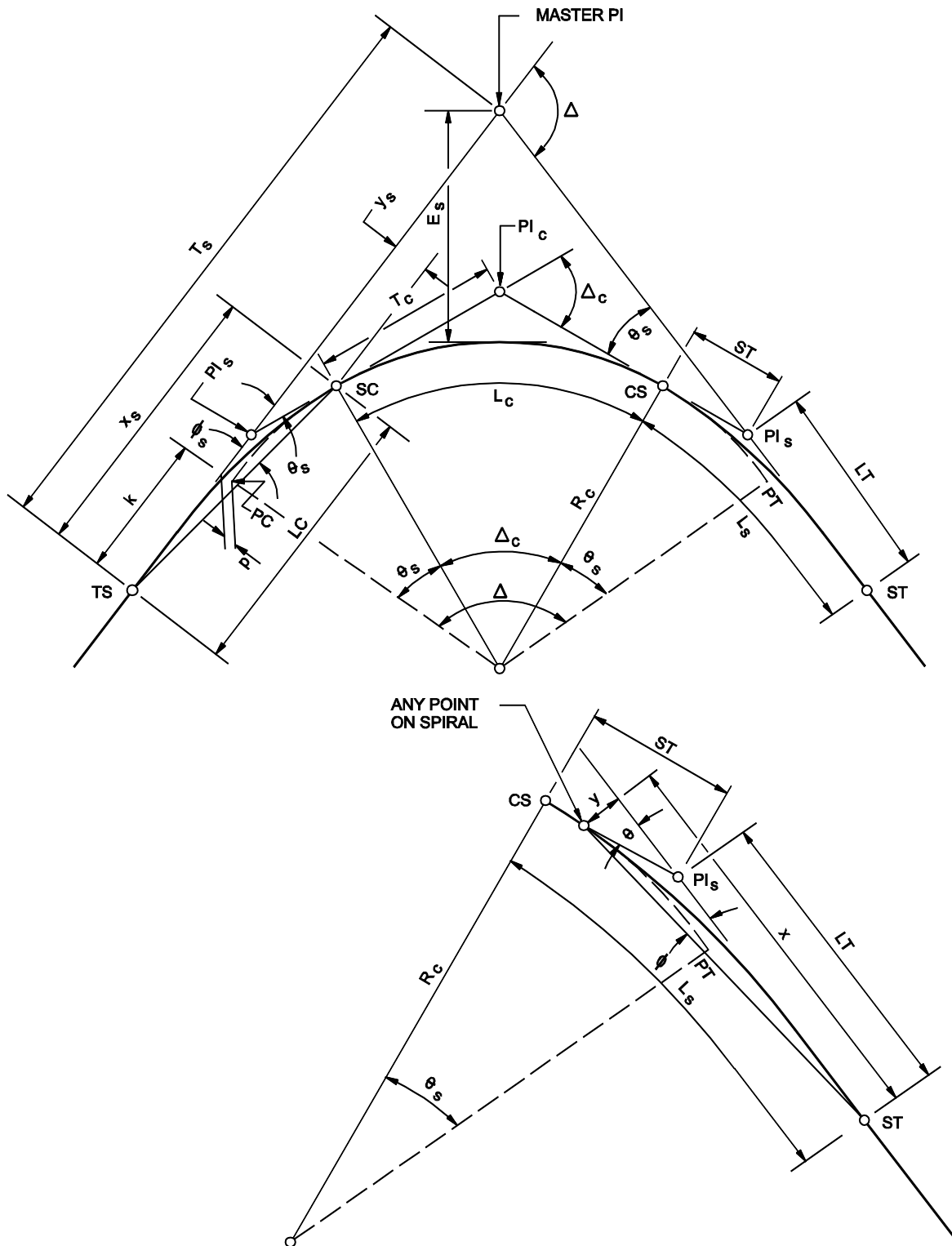
# Horizontal and Vertical Alignment Equations

Appendix H contains additional horizontal and vertical alignment equations that correspond to Chapters 3 and 4, as well as the horizontal and vertical alignment example calculations shown in Appendix K.

### **H.1 SPIRAL CURVES**

The following sections provide information on spiral curve elements, nomenclature and formulas.

**H.1.1 Spiral Curve Elements**



**H.1.2 Spiral Curve Nomenclature**

Master $PI$	=	Point of intersection of the main tangents.	$LC$	=	Long chord of spiral, ft.
$PC$	=	Point at which the circular curve extended becomes parallel to the line from $TS$ to the Master $PI$ .	$p$	=	Offset distance from the main tangent to the $PC$ or $PT$ of the circular curve produced, ft
$PT$	=	Point at which the circular curve extended becomes parallel to the line from $ST$ to the Master $PI$ .	$k$	=	Distance from $TS$ to point on main tangent opposite the $PC$ of the circular curve produced, ft
$PI_c$	=	Point of intersection of circular curve tangents.	$\Delta$	=	Total deflection angle between main tangents of the entire curve, degrees
$PI_s$	=	Point of intersection of the main tangent and tangent of circular curve.	$\Delta_c$	=	Deflection angle between tangents at the $SC$ and the $CS$ or the central angle of the circular curve, degrees
$TS$	=	Tangent to spiral; common point of spiral and near transition.	$\theta_s$	=	Central angle between the tangent of the complete curve and the tangent at the $SC$ ; i.e., the "spiral angle," degrees
$SC$	=	Spiral to curve; common point of spiral and circular curve of near transition.	$\phi_s$	=	Spiral deflection angle from tangents at $TS$ to $SC$ or from $ST$ to $CS$ , degrees
$CS$	=	Curve to spiral; common point of circular curve and spiral of far transition.	$x_s y_s$	=	Coordinates of $SC$ from the $TS$ or of $CS$ from $ST$ .
$ST$	=	Spiral to tangent; common point of spiral and tangent of far transition.	$L$	=	Length of spiral arc from the $TS$ or $ST$ to any point on the spiral, ft
$R_c$	=	Radius of the circular curve ( $SC$ to $CS$ ), ft	$x, y$	=	Coordinates to any point on the spiral from $TS$ or $ST$ .
$L_s$	=	Length of spiral, ft	$\phi$	=	Spiral deflection angle from $TS$ or $ST$ to any point on spiral, degrees
$L_c$	=	Length of circular curve, ft	$\theta$	=	The central angle of spiral arc $L$ to any point on the spiral, degrees. $\theta$ equals $\theta_s$ when $L$ equals $L_s$ .
$T_s$	=	Tangent distance from Master $PI$ to $TS$ or $ST$ , ft			
$T_c$	=	Tangent distance from $SC$ or $CS$ to $PI_c$ , ft			
$E_s$	=	External distance from Master $PI$ to midpoint of circular curve, ft			
$LT$	=	Long tangent of spiral only, ft			
$ST$	=	Short tangent of spiral only, ft			

**H.1.3 Spiral Curve Formulas****CURVE EQUATIONS**

1.  $\theta_s = (L_s / R_C)(90 / \pi)$

2.  $\Delta_C = \Delta - 2\theta_s$

3.  $L_C = \frac{\Delta_C}{360} 2\pi R_C$

4.  $T_S = (R_C + p)\tan(\Delta/2) + k$

5.  $E_S = (R_C + p)(1/\cos(\Delta/2) - 1) + p = \left[ \frac{(R_C + p)}{\cos(\Delta/2)} - (R_C + p) \right] + p$

6.  $p$  and  $k$  are obtained from *Route Location and Design*, Hickerson (pg. 375).

$$p = L_s \left[ 0.0014544 \theta_s \right] - 1.582315 \theta_s^3 \times (10)^{-8} + 1.022426 \theta_s^5 \times (10)^{-13} - \dots$$

$$k = L_s \left[ 0.5 - 5.076957 \theta_s^2 \times (10)^{-6} + 4.295915 \theta_s^4 \times (10)^{-11} - \dots \right]$$

**SPIRAL EQUATIONS**

Correction for C in Formula : $\varphi = \frac{\theta}{3} - C$								
$\theta_s$ in Degrees	15	20	25	30	35	40	45	50
C in Minutes	0.2	0.4	0.8	1.4	2.2	3.4	4.8	6.6

7.  $\varphi(\text{approx.}) = \frac{\theta}{3}$ , if  $\theta_s < 15^\circ 00'$

8.  $\varphi(\text{approx.}) = \frac{\theta}{3} - C$ , if  $\theta_s \geq 15^\circ 00'$

9.  $\varphi = \frac{\theta_s}{3} \left[ \frac{L}{L_s} \right]^2$

10. Exact value of  $\varphi$  by coordinates

$$\tan \varphi = \frac{y}{x}$$

11.  $ST = \frac{y_s}{\sin \theta_s}$

12.  $LT = x_s - \left( \frac{y_s}{\tan \theta_s} \right)$

13.  $LC = \frac{x_s}{\cos \varphi_s}$

14.  $x_s = LC \cos \varphi_s$

15.  $y_s = LC \sin \varphi_s$

16.  $\theta = \frac{L^2}{L_s^2} \theta_s$

17.  $x = L \left( 1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \frac{\theta^6}{9,360} + \frac{\theta^8}{685,440} \right)^*$

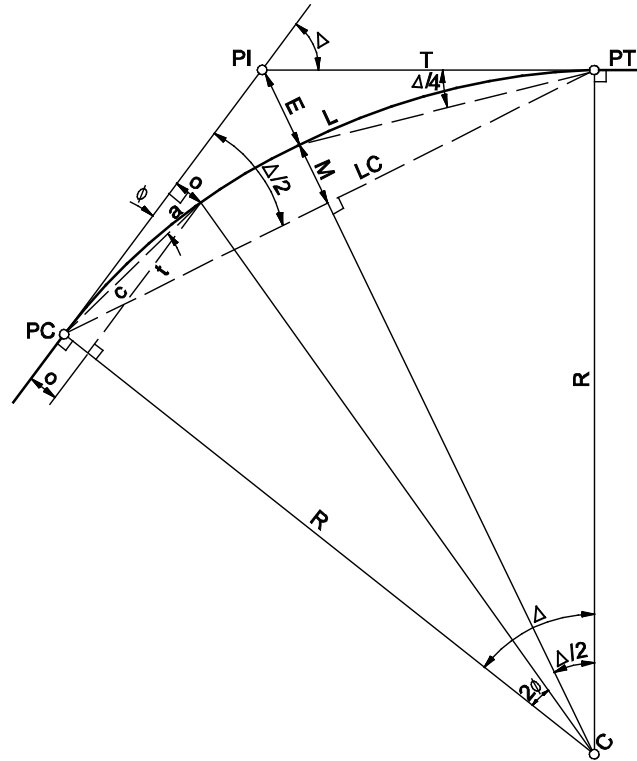
18.  $y = L \left( \frac{\theta}{3} - \frac{\theta^3}{42} + \frac{\theta^5}{1320} - \frac{\theta^7}{75,600} + \frac{\theta^9}{6,894,720} \right)^*$

\*  $\theta$  is in radians for equations 17 and 18 only.Note : These equations are based on *Transitions Curves for Highways* by Barnett.

## H.2 CIRCULAR CURVES

The following sections provide information on circular curve elements, nomenclature and formulas.

### H.2.1 Circular Curve Elements



### H.2.2 Circular Curve Nomenclature

$\Delta$	=	Deflection angle, degrees
$T$	=	Tangent distance, ft. $T$ = distance from $PC$ to $PI$ or distance from $PI$ to $PT$
$L$	=	Length of curve, ft. $L$ = distance from $PC$ to $PT$ along curve
$R$	=	Radius of curvature, ft
$E$	=	External distance ( $PI$ to mid-point of curve), ft
$C$	=	Intersection of radii at center of circular arc
$LC$	=	Length of long chord ( $PC$ to $PT$ ), ft
$M$	=	Middle ordinate (mid-point of arc to mid-point of long chord), ft
$a$	=	Length of arc to any point on a curve, ft
$c$	=	Length of chord from $PC$ to any point on curve, ft
$\phi$	=	Deflection angle from tangent to any point on curve, degrees
$t$	=	Distance along tangent from $PC$ to any point on curve, ft
$o$	=	Tangent offset to any point on curve, ft

**H.2.3 Circular Curve Formulas**

$$T = R(\tan(\Delta/2)) = R \frac{\sin(\Delta/2)}{\cos(\Delta/2)}$$

$$L = \frac{\Delta}{360} 2\pi R$$

$$E = \frac{R}{\cos(\Delta/2)} - R = T \tan(\Delta/4)$$

$$LC = 2R(\sin(\Delta/2)) = 2T(\cos \Delta/2)$$

$$M = R(1 - \cos(\Delta/2)) = E \cos(\Delta/2)$$

$$a = \frac{(200\phi)(2\pi R)}{100(360)} = \frac{(\phi)(\pi R)}{90}$$

$$c = 2R \left( \sin \frac{(100)(360a)}{(200)(2\pi R)} \right) = 2R \left( \sin \frac{90a}{\pi R} \right)$$

$$\phi = \frac{90a}{(\pi R)}$$

$$\cos \phi = (R - o)/2R$$

$$t = R \sin 2\phi = (c) \cos \phi$$

$$o = (c) \sin \phi$$

$$o = R - \sqrt{R^2 - t^2}$$

$$o = R - (R \cos 2\phi)$$

$$o = R(1 - \cos 2\phi)$$

$$\pi = 3.141592654$$

CIRCULAR CURVE ABBREVIATIONS

<i>PC</i>	=	Point of Curvature (Beginning of Curve)
<i>PT</i>	=	Point of Tangency (End of Curve)
<i>PI</i>	=	Point of Intersection of Tangents
<i>PRC</i>	=	Point of Reverse Curvature
<i>PCC</i>	=	Point of Compound Curvature

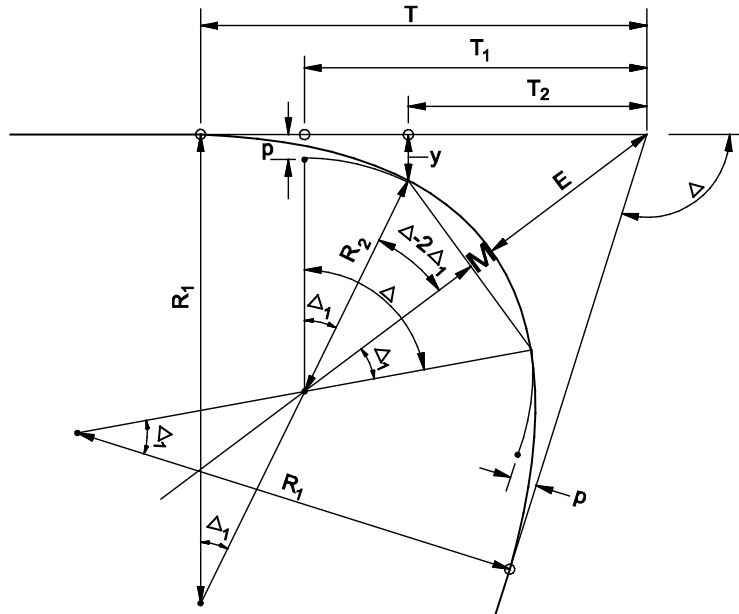
LOCATING THE PC AND PT

Station <i>PC</i>	=	Station <i>PI</i> - <i>T</i>
Station <i>PT</i>	=	Station <i>PC</i> + <i>L</i>
Stations are in 100 feet. For example,		
Sta 13+54.86	means	1,354.86 feet from
Sta 0+00.		

### H.3 COMPOUND CURVES

The following sections provide information on compound curve elements, nomenclature and formulas.

#### H.3.1 Compound Curve Elements



#### H.3.2 Compound Curve Formulas

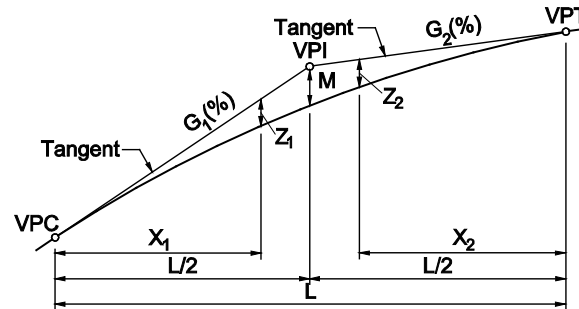
1.  $T_1 = (R_2 + p) \tan \frac{\Delta}{2}$
2.  $\Delta_1 = \cos^{-1} \left[ \frac{R_1 - R_2 - p}{R_1 - R_2} \right]$
3.  $T = T_1 + (R_1 - R_2) \sin \Delta_1$
4.  $T_2 = T_1 - R_2 \sin \Delta_1$
5.  $E = \frac{R_2 + p}{\cos(\Delta/2)} - R_2$
6.  $M = R_2 - [R_2 \cos(\Delta/2 - \Delta_1)]$
7.  $y = (R_2 + p) - R_2 \cos \Delta_1$

Note: "p" is the offset location between the interior curve (extended) to a point where it becomes parallel with the tangent line. See Appendices H.5 and H.6 for other circular curve nomenclature and formulas.

## H.4 SYMMETRICAL VERTICAL CURVES

The following sections provide information on symmetrical vertical curve elements, nomenclature and formulas.

### H.4.1 Symmetrical Vertical Curve Elements



### H.4.2 Symmetrical Vertical Curve Nomenclature

ELEMENT	ABBREVIATION	DEFINITION
Vertical Point of Curvature	<i>VPC</i>	The point at which a tangent grade ends and the vertical curve begins.
Vertical Point of Tangency	<i>VPT</i>	The point at which the vertical curve ends and the tangent grade begins.
Vertical Point of Intersection	<i>VPI</i>	The point where the extension of two tangent grades intersect.
Grade	$G_1, G_2$	The rate of slope between two adjacent VPI's expressed as a percent. The numerical value for percent of grade is the vertical rise or fall in feet for each 100' of horizontal distance. Upgrades in the direction of stationing are identified as plus (+). Downgrades are identified as minus (-).
External Distance	<i>M</i>	The vertical distance (offset) between the VPI and the roadway surface along the vertical curve.
Algebraic Difference in Grade	<i>A</i>	The value of <i>A</i> is determined by the deflection in percent between two tangent grades ( $G_2 - G_1$ ).
Length of Vertical Curve	<i>L</i>	The horizontal distance in feet from the VPC to the VPT.
Tangent Elevation	<i>TAN. ELEV.</i>	The elevation on the tangent line between the VPC and VPI and the VPI and VPT.
Elevation on Vertical Curve	<i>CURVE ELEV.</i>	The elevation of the vertical curve at any given point along the curve.
Horizontal Distance	<i>X</i>	Horizontal distance measured from the VPC or VPT to any point on the vertical curve, in feet.
Tangent Offset	<i>Z</i>	Vertical distance from the tangent line to any point on the vertical curve, in feet.
Low/High Point	$X_T$	The station at the high point for crest curves or the low point for sag curves.
Symmetrical Curve	—	The VPI is located at mid-point between VPC and VPT stationing.
Unsymmetrical Curve	—	The VPI is <u>not</u> located at mid-point between VPC and VPT stationing.



**H.4.3 Symmetrical Vertical Curve Formulas**

Note: The variables  $G_1$  and  $G_2$  are percent values.

**Elevations of VPC and VPT:**

$$ELEV. \text{ OF VPC} = ELEV. \text{ VPI} - G_1 \left( \frac{L}{200} \right)$$

$$ELEV. \text{ OF VPT} = ELEV. \text{ VPI} + G_2 \left( \frac{L}{200} \right)$$

**For the elevation of any point "X" on the vertical curve:**

$$CURVE \text{ ELEV.} = TAN \text{ ELEV.} + Z$$

where:

Left of VPI ( $X_1$  measured from VPC):

$$(a) \text{ TAN. ELEV.} = VPC \text{ ELEV.} + G_1 \left( \frac{X_1}{100} \right)$$

$$(b) Z_1 = X_1^2 \frac{(G_2 - G_1)}{200L}$$

Right of VPI ( $X_2$  measured from VPT):

$$(a) \text{ TAN. ELEV.} = VPT \text{ ELEV.} - G_2 \left( \frac{X_2}{100} \right)$$

$$(b) Z_2 = X_2^2 \frac{(G_2 - G_1)}{200L}$$

**Calculating high or low point in the vertical curve:**

$$(a) \text{ To determine distance " } X_T \text{ " from VPC: } X_T = \frac{L G_1}{G_1 - G_2}$$

$$(b) \text{ To determine high or low point stationing: } VPC \text{ Sta.} + X_T$$

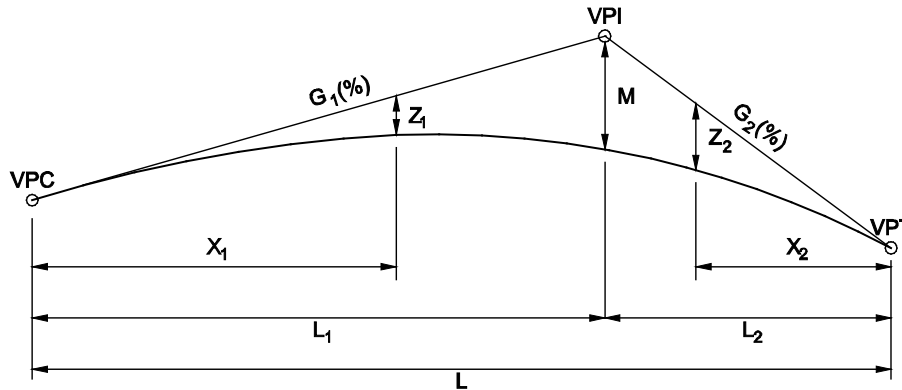
(c) To determine high or low point elevation on the vertical curve:

$$ELEV. \text{ HIGH OR LOW POINT} = ELEV. \text{ VPC} - \frac{L G_1^2}{(G_2 - G_1) 200}$$

## H.5 ASYMMETRICAL VERTICAL CURVES

The following sections provide information on asymmetrical vertical curve elements, nomenclature and formulas.

### H.5.1 Asymmetrical Vertical Curve Elements



### H.5.2 Asymmetrical Vertical Curve Nomenclature

- $M$  = Offset from the VPI to the curve (external distance), feet
- $Z$  = Any tangent offset, feet
- $L$  = Horizontal length of vertical curve, feet
- $L_1$  = Horizontal distance from VPC to VPI, feet
- $L_2$  = Horizontal distance from VPI to VPT, feet
- $X$  = Horizontal distance from VPC or VPT to any ordinate "Z," feet.
- $G_1$  &  $G_2$  = Rates of grade, expressed algebraically, percent.

NOTE: ALL EXPRESSIONS TO BE CALCULATED ALGEBRAICALLY (Use algebraic signs of grades; grades in percent.)

### H.5.3 Asymmetrical Vertical Curve Formulas

1. Elevations of VPC and VPT:

$$ELEV. \text{ OF } VPC = ELEV. \text{ VPI} - G_1 \left( \frac{L_1}{100} \right)$$

$$ELEV. \text{ OF } VPT = ELEV. \text{ VPI} + G_2 \left( \frac{L_2}{100} \right)$$

2. For the elevation of any point “X” on the vertical curve:

$$CURVE \ ELEV. = TAN. \ ELEV. + Z$$

Where:

Left of VPI ( $X_1$  measured from VPC):

$$(a) \ TAN. \ ELEV. = VPC \ ELEV. + G_1 \left( \frac{X_1}{100} \right)$$

$$(b) \ Z_1 = X_1^2 \left( \frac{L_2}{L_1} \right) \left( \frac{G_2 - G_1}{200 \ L} \right)$$

Right of VPI ( $X_2$  measured from VPT):

$$(a) \ TAN. \ ELEV. = VPT \ ELEV. - G_2 \left( \frac{X_2}{100} \right)$$

$$(b) \ Z_2 = X_2^2 \left( \frac{L_1}{L_2} \right) \left( \frac{G_2 - G_1}{200 \ L} \right)$$

3. Calculating High or Low Point on Curve:

Note: Two answers will be determined by solving the equations below. Only one answer is correct. The incorrect answer is where  $X_T > L_1$  on the left side of the VPI or where  $X_T > L_2$  on the right side of the VPI.

- a. Assume high or low point occurs left of VPI to determine the distance,  $X_T$ , from VPC:

$$X_T = \frac{L_1}{L_2} \left[ \frac{G_1 L}{(G_1 - G_2)} \right]$$

Note: Is  $X_T > L_1$ ? If yes, this answer is incorrect and the high or low point is on the right side of the VPI. (Go to step “d.” to solve for the high or low point elevation.) If no, then this is the correct answer and proceed with steps b. and c. below.)

- b. To determine high or low point stationing (where  $X_T < L_1$ ):

$$STA_{HIGH \ OR \ LOW \ POINT} = VPC \ STA. + X_T$$

- c. To determine high or low point elevation on vertical curve (when  $X_T < L_1$ ):

$$ELEV. \ HIGH \ OR \ LOW \ POINT = ELEV. \ VPC - \frac{L_1}{L_2} \left[ \frac{L G_1^2}{(G_2 - G_1) 200} \right]$$

- d. If  $X_T > L_1$  from step a., the high or low point occurs right of the VPI. Determine the distance  $X_T$  from the VPT:

$$X_T = \frac{L_2}{L_1} \left[ \frac{G_2 L}{(G_2 - G_1)} \right]$$

- e. To determine high or low point stationing:

$$STA_{HIGH OR LOW POINT} = VPT STA. - X_T$$

- f. To determine high or low point elevation on the vertical curve:

$$ELEV. HIGH OR LOW POINT = ELEV. VPT - \frac{L_2}{L_1} \left[ \frac{LG_2^2}{(G_2 - G_1)200} \right]$$