Appendix H

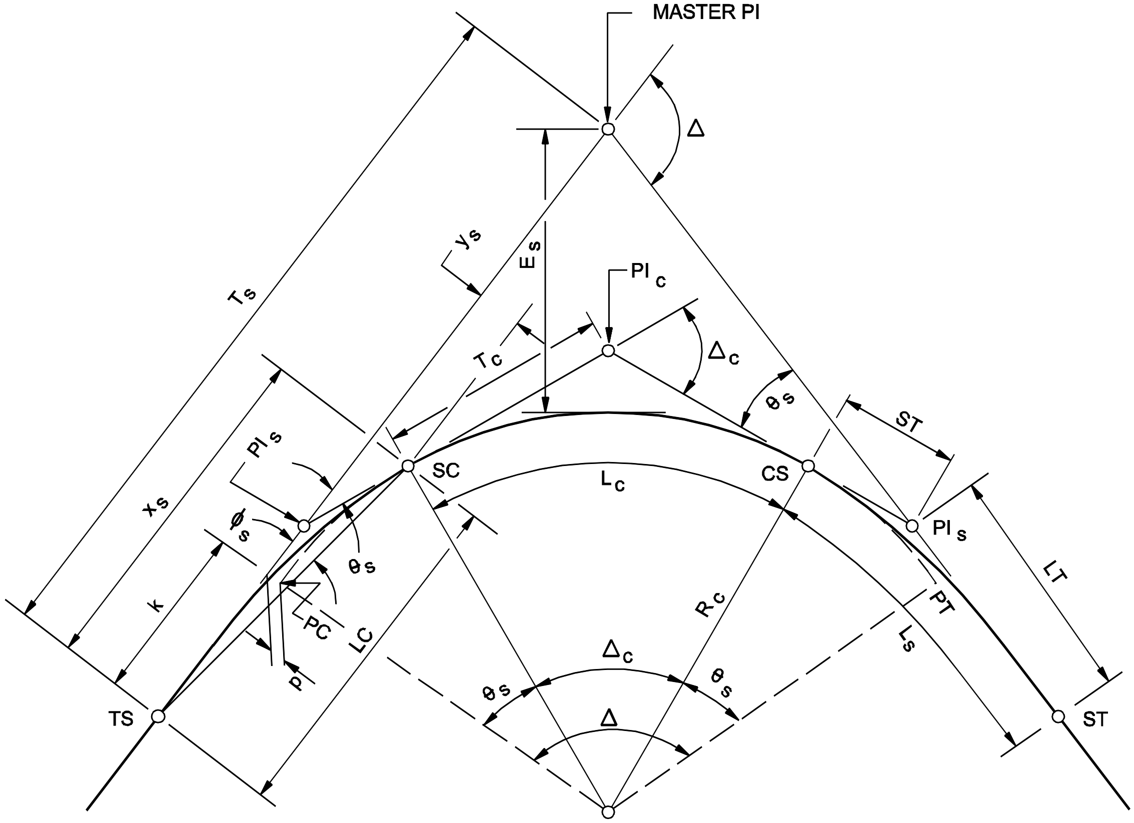
Horizontal and Vertical Alignment Equations

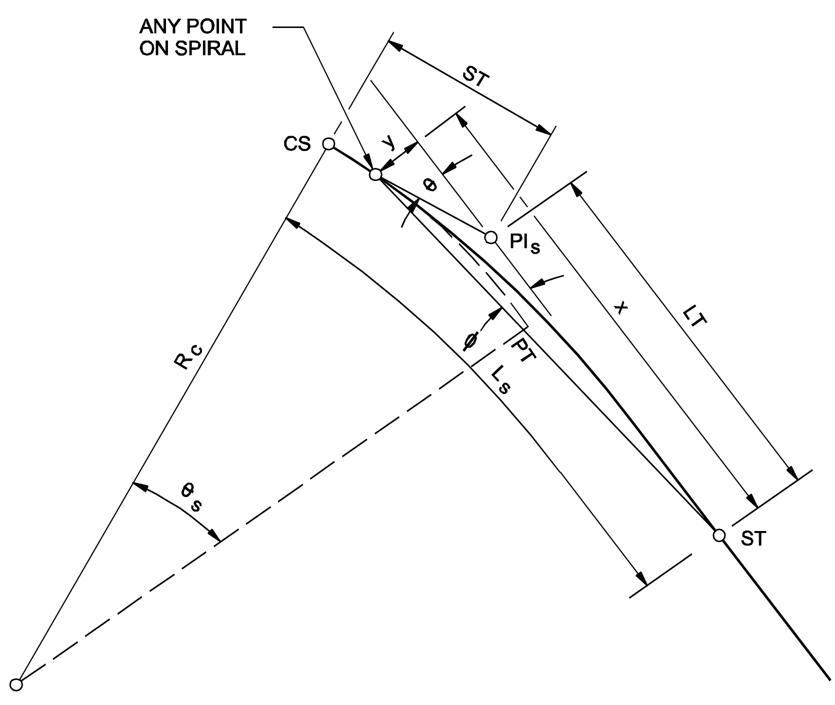
Appendix H contains additional horizontal and vertical alignment equations that correspond to Chapters 3 and 4, as well as the horizontal and vertical alignment example calculations shown in Appendix K.

H.1 SPIRAL CURVES

The following sections provide information on spiral curve elements, nomenclature and formulas.

H.1.1 Spiral Curve Elements





H.1.2 Spiral Curve Nomenclature

Master *PI*	=	Point of intersection of the main tangents.
PC	=	Point at which the circular curve extended becomes parallel to the line from *TS* to the Master *PI.*
PT	=	Point at which the circular curve extended becomes parallel to the line from *ST* to the Master *PI.*
PI_c	=	Point of intersection of circular curve tangents.
PI_s	=	Point of intersection of the main tangent and tangent of circular curve.
TS	=	Tangent to spiral; common point of spiral and near transition.
SC	=	Spiral to curve; common point of spiral and circular curve of near transition.
CS	=	Curve to spiral; common point of circular curve and spiral of far transition.
ST	=	Spiral to tangent; common point of spiral and tangent of far transition.
R_c	=	Radius of the circular curve (*SC* to *CS*), ft
L_s	=	Length of spiral, ft
L_c	=	Length of circular curve, ft
T_s	=	Tangent distance from Master *PI* to *TS* or *ST*, ft
T_c	=	Tangent distance from *SC* or *CS* to PI_c, ft
E_s	=	External distance from Master *PI* to midpoint of circular curve, ft
LT	=	Long tangent of spiral only, ft
ST	=	Short tangent of spiral only, ft
LC	=	Long chord of spiral, ft.
p	=	Offset distance from the main tangent to the *PC* or *PT* of the circular curve produced, ft
k	=	Distance from *TS* to point on main tangent opposite the *PC* of the circular curve produced, ft
Δ	=	Total deflection angle between main tangents of the entire curve, degrees
Δ_c	=	Deflection angle between tangents at the *SC* and the *CS* or the central angle of the circular curve, degrees
θ_s	=	Central angle between the tangent of the complete curve and the tangent at the *SC;* i.e., the "spiral angle," degrees
ϕ_s	=	Spiral deflection angle from tangents at *TS* to *SC* or from *ST* to *CS*, degrees
$x_s y_s$	=	Coordinates of *SC* from the *TS* or of *CS* from *ST.*
L	=	Length of spiral arc from the *TS* or *ST* to any point on the spiral, ft
x,y	=	Coordinates to any point on the spiral from TS or ST.
ϕ	=	Spiral deflection angle from *TS* or *ST* to any point on spiral, degrees
θ	=	The central angle of spiral arc *L* to any point on the spiral, degrees. θ equals θ_s when L equals L_s.

H.1.3 Spiral Curve Formulas

CURVE EQUATIONS

1. $\theta_S = (L_s / R_C)(90/\pi)$
2. $\Delta_C = \Delta - 2\theta_S$
3. $L_C = \frac{\Delta_C}{360} 2\pi R_C$
4. $T_S = (R_C + p)\tan(\Delta/2) + k$
5. $E_S = (R_C + p)(1/\cos(\Delta/2) - 1) + p = \left[\frac{(R_C + p)}{\cos(\Delta/2)} - (R_C + p)\right] + p$

6. p and k are obtained from *Route Location and Design*, Hickerson (pg. 375).

$$p = L_s\left[0.0014544\ \theta_s\right] - 1.582315\ \theta_s^{\ 3} \times (10)^{-8} + 1.022426\ \theta_s^{\ 5} \times (10)^{-13} - \ldots$$

$$k = L_s\left[0.5 - 5.076957\ \theta_s^{\ 2} \times (10)^{-6} + 4.295915\ \theta_s^{\ 4} \times (10)^{-11} - \ldots\right]$$

SPIRAL EQUATIONS

Correction for C in Formula : $\varphi = \frac{\theta}{3} - C$								
θ_S in Degrees	15	20	25	30	35	40	45	50
C in Minutes	0.2	0.4	0.8	1.4	2.2	3.4	4.8	6.6

7. $\varphi(approx.) = \frac{\theta}{3}\ ,\ if\ \ \theta_S < \ 15°00'$
8. $\varphi(approx.) = \frac{\theta}{3} - C,\ if\ \ \theta_S \geq \ 15°00'$
9. $\varphi = \frac{\theta_S}{3}\left[\frac{L}{L_S}\right]^2$
10. Exact value of φ by coordinates

    $\tan\varphi = \frac{y}{x}$
11. $ST = \frac{y_S}{\sin\theta_S}$
12. $LT = x_S - \left(\frac{y_S}{\tan\theta_S}\right)$
13. $LC = \frac{x_s}{\cos\varphi_S}$
14. $x_S = LC\cos\varphi_S$
15. $y_S = LC\sin\varphi_S$
16. $\theta = \frac{L^2}{L_S^{\ 2}}\theta_s$
17. $x = L\left(1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \frac{\theta^6}{9{,}360} + \frac{\theta^8}{685{,}440}\right)^*$
18. $y = L\left(\frac{\theta}{3} - \frac{\theta^3}{42} + \frac{\theta^5}{1320} - \frac{\theta^7}{75{,}600} + \frac{\theta^9}{6{,}894{,}720}\right)^*$

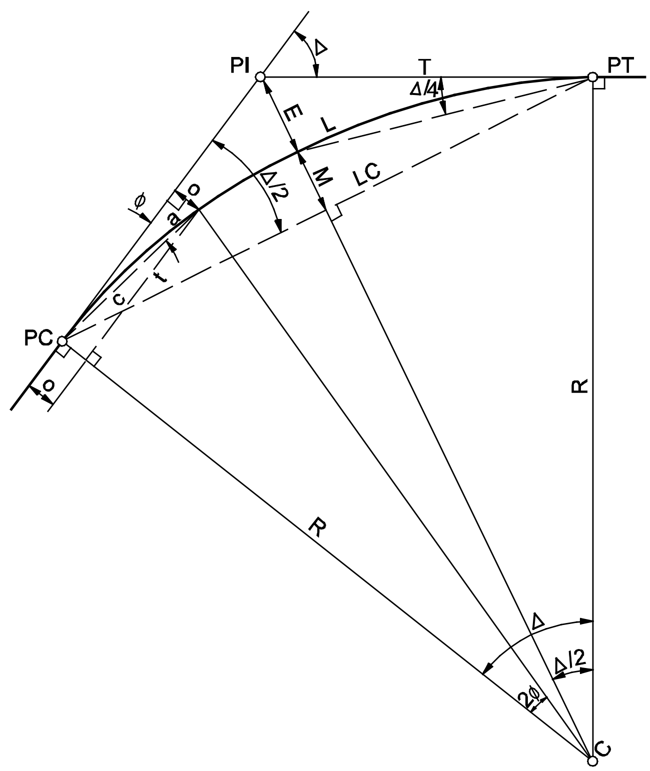
* *θ is in radians for equations 17 and 18 only.*

Note : These equations are based on *Transitions Curves for Highways* by Barnett.

H.2 CIRCULAR CURVES

The following sections provide information on circular curve elements, nomenclature and formulas.

H.2.1 Circular Curve Elements



H.2.2 Circular Curve Nomenclature

Δ	=	Deflection angle, degrees
T	=	Tangent distance, ft. T = distance from *PC* to *PI* or distance from *PI* to *PT*
L	=	Length of curve, ft. L = distance from *PC* to *PT* along curve
R	=	Radius of curvature, ft
E	=	External distance (*PI* to mid-point of curve), ft
C	=	Intersection of radii at center of circular arc
LC	=	Length of long chord (*PC* to *PT*), ft
M	=	Middle ordinate (mid-point of arc to mid-point of long chord), ft
a	=	Length of arc to any point on a curve, ft
c	=	Length of chord from *PC* to any point on curve, ft
φ	=	Deflection angle from tangent to any point on curve, degrees
t	=	Distance along tangent from *PC* to any point on curve, ft
o	=	Tangent offset to any point on curve, ft

H.2.3 Circular Curve Formulas

$$T = R(\tan(\Delta/2)) = R\frac{\sin(\Delta/2)}{\cos(\Delta/2)}$$

$$L = \frac{\Delta}{360}2\pi R$$

$$E = \frac{R}{\cos(\Delta/2)} - R = T\tan(\Delta/4)$$

$$LC = 2R(\sin(\Delta/2)) = 2T(\cos\Delta/2)$$

$$M = R(1-\cos(\Delta/2)) = E\cos(\Delta/2)$$

$$a = \frac{(200\varphi)(2\pi R)}{100(360)} = \frac{(\varphi)(\pi R)}{90}$$

$$c = 2R\left(\sin\frac{(100)(360a)}{(200)(2\pi R)}\right) = 2R(\sin\frac{90a}{\pi R})$$

$$\varphi = \frac{90a}{(\pi R)}$$

$$\cos\varphi = (R - o)/2R$$

$$t = R\sin 2\varphi = (c)\cos\varphi$$

$$o = (c)\sin\varphi$$

$$o = R - \sqrt{R^2 - t^2}$$

$$o = R - (R\cos 2\varphi)$$

$$o = R(1 - \cos 2\varphi)$$

$$\pi = 3.141592654$$

CIRCULAR CURVE ABBREVIATIONS

PC = Point of Curvature (Beginning of Curve)
PT = Point of Tangency (End of Curve)
PI = Point of Intersection of Tangents
PRC = Point of Reverse Curvature
PCC = Point of Compound Curvature

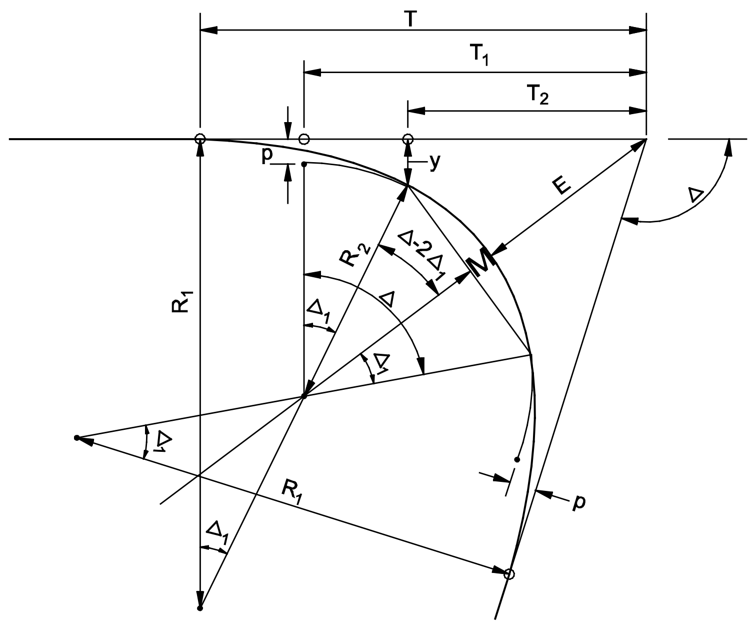
LOCATING THE PC AND PT

Station *PC* = Station *PI* – *T*
Station *PT* = Station *PC* + *L*
Stations are in 100 feet. For example, Sta 13+54.86 means 1,354.86 feet from Sta 0+00.

H.3 COMPOUND CURVES

The following sections provide information on compound curve elements, nomenclature and formulas.

H.3.1 Compound Curve Elements



H.3.2 Compound Curve Formulas

1. $T_1 = (R_2 + p)\tan\frac{\Delta}{2}$

2. $\Delta_1 = \cos^{-1}\left[\frac{R_1 - R_2 - p}{R_1 - R_2}\right]$

3. $T = T_1 + (R_1 - R_2)\sin\Delta_1$

4. $T_2 = T_1 - R_2\sin\Delta_1$

5. $E = \frac{R_2 + p}{\cos(\Delta/2)} - R_2$

6. $M = R_2 - \lfloor R_2\cos(\Delta/2 - \Delta_1)\rfloor$

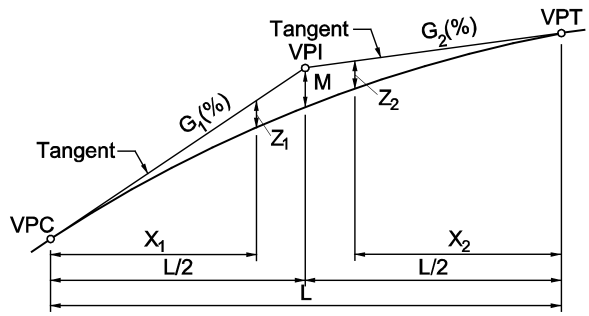
7. $y = (R_2 + p) - R_2\cos\Delta_1$

Note: "p" is the offset location between the interior curve (extended) to a point where it becomes parallel with the tangent line. See Appendices H.5 and H.6 for other circular curve nomenclature and formulas.

H.4 SYMMETRICAL VERTICAL CURVES

The following sections provide information on symmetrical vertical curve elements, nomenclature and formulas.

H.4.1 Symmetrical Vertical Curve Elements



H.4.2 Symmetrical Vertical Curve Nomenclature

ELEMENT	ABBREVIATION	DEFINITION
Vertical Point of Curvature	*VPC*	The point at which a tangent grade ends and the vertical curve begins.
Vertical Point of Tangency	*VPT*	The point at which the vertical curve ends and the tangent grade begins.
Vertical Point of Intersection	*VPI*	The point where the extension of two tangent grades intersect.
Grade	G_1, G_2	The rate of slope between two adjacent VPI's expressed as a percent. The numerical value for percent of grade is the vertical rise or fall in feet for each 100' of horizontal distance. Upgrades in the direction of stationing are identified as plus (+). Downgrades are identified as minus (-).
External Distance	M	The vertical distance (offset) between the VPI and the roadway surface along the vertical curve.
Algebraic Difference in Grade	A	The value of *A* is determined by the deflection in percent between two tangent grades ($G_2 - G_1$).
Length of Vertical Curve	L	The horizontal distance in feet from the VPC to the VPT.
Tangent Elevation	*TAN. ELEV.*	The elevation on the tangent line between the VPC and VPI and the VPI and VPT.
Elevation on Vertical Curve	*CURVE ELEV.*	The elevation of the vertical curve at any given point along the curve.
Horizontal Distance	X	Horizontal distance measured from the *VPC* or *VPT* to any point on the vertical curve, in feet.
Tangent Offset	Z	Vertical distance from the tangent line to any point on the vertical curve, in feet.
Low/High Point	X_T	The station at the high point for crest curves or the low point for sag curves.
Symmetrical Curve	—	The *VPI* is located at mid-point between *VPC* and *VPT* stationing.
Unsymmetrical Curve	—	The *VPI* is not located at mid-point between *VPC* and *VPT* stationing.

H.4.3 Symmetrical Vertical Curve Formulas

Note: The variables G_1 and G_2 are percent values.

Elevations of *VPC* and *VPT*:

$$ELEV.\ OF\ VPC = ELEV.\ VPI - G_1\left(\frac{L}{200}\right)$$

$$ELEV.\ OF\ VPT = ELEV.\ VPI + G_2\left(\frac{L}{200}\right)$$

For the elevation of any point "*X*" on the vertical curve:

$$CURVE\ ELEV. = TAN\ \ ELEV. + Z$$

where:

Left of VPI (X_1 measured from VPC):

(a) $TAN.\ ELEV. = VPC\ ELEV. + G_1\left(\frac{X_1}{100}\right)$

(b) $Z_1 = X_1^2\frac{(G_2 - G_1)}{200\,L}$

Right of VPI (X_2 measured from VPT):

(a) $TAN\ \ ELEV. = VPT\ ELEV. - G_2\left(\frac{X_2}{100}\right)$

(b) $Z_2 = X_2^2\frac{(G_2 - G_1)}{200\,L}$

Calculating high or low point in the vertical curve:

(a) To determine distance "X_T" from *VPC*: $X_T = \frac{L\,G_1}{G_1 - G_2}$

(b) To determine high or low point stationing: $VPC\ \ Sta. + X_T$

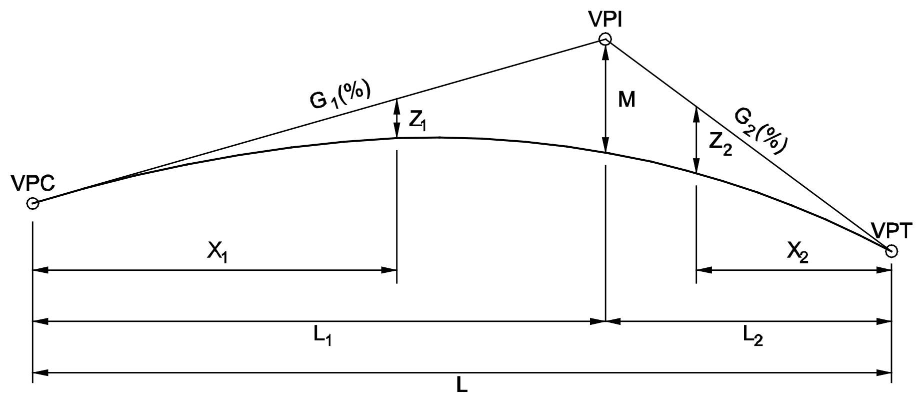
(c) To determine high or low point elevation on the vertical curve:

$$ELEV._{HIGH\ \ OR\ \ LOW\ \ POINT} = ELEV.\ VPC - \frac{L\,G_1^2}{(G_2 - G_1)\,200}$$

H.5 ASYMMETRICAL VERTICAL CURVES

The following sections provide information on asymmetrical vertical curve elements, nomenclature and formulas.

H.5.1 Asymmetrical Vertical Curve Elements



H.5.2 Asymmetrical Vertical Curve Nomenclature

M	=	Offset from the *VPI* to the curve (external distance), feet
Z	=	Any tangent offset, feet
L	=	Horizontal length of vertical curve, feet
L_1	=	Horizontal distance from *VPC* to *VPI*, feet
L_2	=	Horizontal distance from *VPI* to *VPT*, feet
X	=	Horizontal distance from *VPC* or *VPT* to any ordinate "*Z*," feet.
G_1 & G_2	=	Rates of grade, expressed algebraically, percent.

NOTE: ALL EXPRESSIONS TO BE CALCULATED ALGEBRAICALLY (Use algebraic signs of grades; grades in percent.)

H.5.3 Asymmetrical Vertical Curve Formulas

1. Elevations of *VPC* and *VPT*:

$$ELEV.\ OF\ \ VPC = ELEV.\ VPI - G_1\left(\frac{L_1}{100}\right)$$

$$ELEV.\ OF\ \ VPT = ELEV.\ VPI + G_2\left(\frac{L_2}{100}\right)$$

2. For the elevation of any point "X" on the vertical curve:

$$CURVE\ \ ELEV. = TAN.\ ELEV. + Z$$

Where:

<u>Left of *VPI* (X_1 measured from *VPC*)</u>:

(a) $TAN.\ ELEV. = VPC\,ELEV. + G_1\left(\frac{X_1}{100}\right)$

(b) $Z_1 = X_1^2\left(\frac{L_2}{L_1}\right)\left(\frac{G_2 - G_1}{200\ L}\right)$

<u>Right of *VPI* (X_2 measured from *VPT*)</u>:

(a) $TAN.\ ELEV. = VPT\,ELEV. - G_2\left(\frac{X_2}{100}\right)$

(b) $Z_2 = X_2^2\left(\frac{L_1}{L_2}\right)\left(\frac{G_2 - G_1}{200\ L}\right)$

3. Calculating High or Low Point on Curve:

<u>Note</u>: Two answers will be determined by solving the equations below. Only one answer is correct. The incorrect answer is where $X_T > L_1$ on the left side of the *VPI* or where $X_T > L_2$ on the right side of the *VPI*.

a. Assume high or low point occurs left of *VPI* to determine the distance, X_T, from *VPC*:

$$X_T = \frac{L_1}{L_2}\left[\frac{G_1 L}{(G_1 - G_2)}\right]$$

Note: Is $X_T > L_1$? If yes, this answer is incorrect and the high or low point is on the right side of the *VPI*. (Go to step "d." to solve for the high or low point elevation.) If no, then this is the correct answer and proceed with steps b. and c. below.)

b. To determine high or low point stationing (where $X_T < L_1$):

$$STA_{\ HIGH\,OR\,LOW\,POINT} = VPC\,STA. + X_T$$

c. To determine high or low point elevation on vertical curve (when $X_T < L_1$):

$$ELEV._{HIGH\ OR\ LOW\ POINT} = ELEV.\ VPC - \frac{L_1}{L_2}\left[\frac{L G_1^2}{(G_2 - G_1)\,200}\right]$$

d. If $X_T > L_1$ from step a., the high or low point occurs right of the *VPI*. Determine the distance X_T from the *VPT*:

$$X_T = \frac{L_2}{L_1}\left[\frac{G_2 L}{(G_2 - G_1)}\right]$$

e. To determine high or low point stationing:

$$STA_{\ HIGH\ OR\ LOW\ POINT} = VPT\ STA. - X_T$$

f. To determine high or low point elevation on the vertical curve:

$$ELEV._{HIGH\ OR\ LOW\ POINT} = ELEV.\ VPT - \frac{L_2}{L_1}\left[\frac{LG_2^2}{(G_2 - G_1)200}\right]$$